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REMARKS ON THE STABILITY ANALYSIS OF REACTIVE FLOWS

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ABSTRACT

We study a simple model of compressible reacting flow. First, we derive a dispersion relation for the linearized problem, making a distinction between frozen and equilibrium sound speed. Second, we study the stability of the Von Neumann-Richtmyer scheme applied to this model. One finds a natural generalization of the C.F.L. condition.

INTRODUCTION

A simple model--one irreversible reaction--of compressible reactive flow is presented in this report. Equations of gas dynamics are augmented by one equation for the progress variable of the chemical reaction and the appropriate equations of state. The resulting set of equations is a nonlinear hyperbolic system with source terms. However, acoustic waves, i.e., small perturbation of a given constant state, will not be described any more by the wave equation but by a more general dispersive equation. Only in the so-called frozen equilibrium limit will the wave equation be recovered with the appropriate sound speed. Such a classical analysis (see [8],[1] for instance) is extended to the discrete set of equations obtained by using the Von Neumann-Richtmyer scheme (see [9]). Such a scheme is known for its simplicity and its robustness to handle strong shocks. The known stability results (see [9],[7]) for such a scheme are generalized in our context. Both frozen and equilibrium limits appear again, and we make a distinction between implicit and explicit schemes used to discretize the reaction equation.

1. THE DISPERSION RELATION FOR A REACTIVE FLOW

We consider 1-D reactive flows involving a single irreversible chemical reaction $A \rightarrow B$. The flows will be compressible, viscous, but heat conduction and chemical diffusion effects are neglected.

Let us introduce the pressure p, the specific volume v, the internal energy e, the velocity u, and the progress variable λ of the reaction $A \rightarrow B$, changing from 0 for no reaction to 1 for complete reaction. Using Lagrangian coordinates (a,t), equations of motion are given by:

$$\begin{cases} \dfrac{\partial v}{\partial t} - \dfrac{\partial u}{\partial a} = 0 \\[3mm] \dfrac{\partial u}{\partial t} + \dfrac{\partial \overline{p}}{\partial a} = 0 \\[3mm] \dfrac{\partial}{\partial t}\left(e + \dfrac{u^2}{2}\right) + \dfrac{\partial}{\partial a}(\overline{p}u) = 0 \\[3mm] \dfrac{\partial \lambda}{\partial t} = r. \end{cases}$$

(1.1)

We recall that (a,t) are related to the Eulerian coordinates (x,t), where $x = x(a,t)$ gives the position, at time t, of a fluid element that was initially at position a. We complete the system (1.1) by prescribing $\overline{p}$, the internal energy (per unit mass) e, and the reaction rate r; precisely we have

(1.2)
$$\overline{p} = p + \omega = p - \mu \frac{\partial u}{\partial a} \qquad\qquad \mu > 0$$

where pr is the artificial viscosity

(1.3)
$$e = e(p,v,\lambda)$$

(1.4)
$$r = r(p,v,\lambda).$$

In (1.1), we rewrite the third equation in a nonconservative form as

(1.5)
$$\frac{\partial e}{\partial t} + \overline{p}\,\frac{\partial v}{\partial t} = 0.$$

Using (1.1),(1.3) and (1.5), we also obtain

(1.6)
$$\frac{\partial e}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial e}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial e}{\partial \lambda}\frac{\partial \lambda}{\partial t} + \bar{p}\frac{\partial v}{\partial t} = 0,$$

hence a relation between $\frac{\partial p}{\partial t}$ and $\frac{\partial v}{\partial t}$

(1.7)
$$\frac{\partial e}{\partial p}\left(\frac{\partial p}{\partial t} + \frac{\frac{\partial e}{\partial v} + \bar{p}}{\frac{\partial e}{\partial p}}\frac{\partial v}{\partial t} + \frac{\frac{\partial e}{\partial \lambda}}{\frac{\partial e}{\partial p}}r\right) = 0.$$

We first notice (see [3] for instance) that

(1.8)
$$\frac{\frac{\partial e}{\partial \lambda}}{\frac{\partial e}{\partial p}} = -\left(\frac{\partial p}{\partial \lambda}\right)_{e,v}.$$

Next, we set

(1.9)
$$\bar{a}^2 = \frac{\frac{\partial e}{\partial v} + \bar{p}}{\frac{\partial e}{\partial p}}$$

(1.10)
$$\bar{b} = \left(\frac{\partial p}{\partial \lambda}\right)_{e,v}.$$

The first coefficient will be interpreted (see the Appendix below) as the mass speed of sound; the second one is related to the themicity (see [3]). Therefore, (1.7) reads

(1.11)
$$\frac{\partial p}{\partial t} + \bar{a}^2\frac{\partial v}{\partial t} = \bar{b}r.$$

In order to investigate the stability of the solution of (1.1)-(1.4), we shall "linearize" the equation for λ around some local equilibrium state $\lambda^* = \lambda^*(p,v)$ (see the Appendix). We have

$$(1.12) \qquad \frac{\partial \lambda}{\partial t} = \frac{\lambda^*(p,v) - \lambda}{\tau} \, ,$$

where τ is a "relaxation time," assumed given. Next, we linearize the equilibrium state by taking the undisturbed fluid span--denoted by the subscript 0--as reference state

$$(1.13) \qquad \lambda^*(p,v) = \lambda^*_{p_0} p + \lambda^*_{v_0} v.$$

Finally, we obtain for the equation governing λ

$$(1.14) \qquad \frac{\partial \lambda}{\partial t} = \frac{1}{\tau} (\lambda^*_{p_0} p + \lambda^*_{v_0} v - \lambda).$$

The linearized system associated to (1.1) is obtained classically by writing $v = v_0 + v'$, $p = p_0 + p'$, etc., in (1.1),(1.11), and (1.14), where $v_0, p_0, \cdots$ is a constant reference state. Neglecting second order terms and performing the approximation of frozen coefficients, we finally find (dropping the $'$)

$$(1.15) \qquad \begin{cases} \dfrac{\partial v}{\partial t} - \dfrac{\partial u}{\partial a} = 0 \\[2mm] \dfrac{\partial u}{\partial t} + \dfrac{\partial p}{\partial a} = \dfrac{\partial}{\partial a} (\bar{\mu} \dfrac{\partial u}{\partial a}) \\[2mm] \dfrac{\partial p}{\partial t} + \bar{a}^2 \dfrac{\partial v}{\partial t} = \bar{b} \dfrac{\partial \lambda}{\partial t} \\[2mm] \dfrac{\partial \lambda}{\partial t} = \dfrac{1}{\tau} (\lambda^*_{p_0} p + \lambda^*_{v_0} v - \lambda), \end{cases}$$

where $\bar{\mu}, \bar{a}^2, \bar{b}, \tau, \lambda^*_{p_0}, \lambda^*_{v_0}$ are now given constants. An equivalent form of (1.15), which is more convenient for our analysis, is

$$(1.16) \quad \begin{cases} \dfrac{\partial v}{\partial t} - \dfrac{\partial u}{\partial a} = 0 \\[2mm] \dfrac{\partial u}{\partial t} + \dfrac{\partial p}{\partial a} = \dfrac{\partial}{\partial a}\left(\bar{\mu}\,\dfrac{\partial u}{\partial a}\right) \\[2mm] \dfrac{\partial p}{\partial t} + \bar{a}^2\,\dfrac{\partial u}{\partial a} = \bar{b}\,\dfrac{\partial \lambda}{\partial t} \\[2mm] \dfrac{\partial \lambda}{\partial t} = \dfrac{1}{\tau}\left(\lambda^*_{p_0}\,p + \lambda^*_{v_0}\,v - \lambda\right). \end{cases}$$

The dispersion relation associated with (1.16) is classically obtained (see [10] for instance) by representing v, u, p, λ in (1.16) as a harmonic

$$(1.17) \qquad v = V_0 \exp(\omega t + ika) \ldots, \text{ etc.}$$

Substituting (1.17) into (1.16), we obtain a linear system for V_0, U_0, P_0, l_0

$$(1.18) \quad \begin{cases} \omega V_0 - ik U_0 = 0 \\[2mm] \omega U_0 + ik P_0 = -\bar{\mu}\, k^2 U_0 \\[2mm] \omega P_0 + \bar{a}^2\, ik U_0 = \bar{b}\, \omega\, l_0 \\[2mm] \omega l_0 = \dfrac{1}{\tau}\left(\lambda^*_{p_0}\,P_0 + \lambda^*_{v_0}\,V_0 - l_0\right). \end{cases}$$

To have a nontrivial solution, the determinant associated with (1.18) should be zero, and this gives the dispersion relation, a relation between ω and k. A simple computation gives

(1.19)
$$\omega[\tau\omega(\omega^2 + \overline{\mu} \, k^2\omega + \overline{a}^2 \, k^2) + (1 - \overline{b} \, \lambda^*_{p_0})(\omega^2 + \overline{\mu} \, k^2\omega) +$$

$$(\overline{a}^2 - \overline{b} \, \lambda^*_{v_0})k^2] = 0.$$

We rearrange this equation as

(1.20)
$$\omega[\frac{\tau \, \overline{a}^2}{\overline{a}^2 - \overline{b}\lambda^*_{v_0}} \, \omega(\frac{1}{\overline{a}^2} \, \omega^2 + k^2 + \frac{\overline{\mu}}{\overline{a}^2} \, k^2\omega)$$

$$+ \frac{1 - \overline{b} \, \lambda^*_{p_0}}{\overline{a}^2 - \overline{b} \, \lambda^*_{v_0}} \, \omega^2 + k^2 + \frac{1 - \overline{b} \, \lambda^*_{p_0}}{\overline{a}^2 - \overline{b} \, \lambda^*_{v_0}} \, \overline{\mu} \, k^2\omega] = 0.$$

Of course the roots $\omega = \omega(k)$ of (1.20) characterize the stability of (1.16) according to the sign of their real parts. In view of the results given in the Appendix, the coefficients in (1.20) do have a simple interpretation in terms of the frozen and equilibrium mass sound speed $\overline{a}_f$ and $\overline{a}_e$. We refer to the Appendix for the definition; here we just note that

(1.21)
$$\overline{a}_f^2 \geq \overline{a}_e^2.$$

We also introduce a new relaxation time:

(1.22)
$$\overline{\tau} = \frac{\overline{a}_f^2}{\overline{a}_f^2 - \overline{b} \, \lambda^*_{v_0}} \, \tau.$$

Finally, we rewrite the characteristic equation (1.20) in terms of $\bar{\tau}$, $\bar{a}_f^2$, and $\bar{a}_e^2$

(1.23)

$$\omega\left[\bar{\tau}\ \omega(\frac{1}{\bar{a}_f^2}\ \omega^2 + k^2 + \frac{\bar{\mu}}{\bar{a}_f^2}\ k^2\omega) \right.$$

$$\left. + \frac{1}{\bar{a}_e^2}\ \omega^2 + k^2 + \frac{\bar{\mu}}{\bar{a}_e^2}\ k^2\omega\right] = 0.$$

Of course, as $\bar{\tau} \to \infty$ (resp. $\bar{\tau} \to 0$), one recovers the limit of the frozen (resp. equilibrium) flow. Both limits are dispersion relations associated to a wave operator perturbed by a damping term $\Delta \frac{\partial u}{\partial t}$, with the appropriate coefficient, corresponding to the viscous term in (1.2).

Due to (1.21), one can easily check, using Routh-Hurwicz criterium (see [5] for instance and the next section)

(1.24) $\mathrm{Re}\ \omega(k) \le 0,$

where $\omega = \omega(k)$ is any root of (1.23). Therefore, the solutions of (1.16) decay with time, i.e., the solutions of (1.1) are linearly stable.

2. VON NEUMANN-RICHTMYER SCHEME FOR REACTIVE FLOWS

Now we investigate the (linear) stability of the Von Neumann-Richtmyer scheme used to discretize (1.1) or (1.16) and completed by a simple Euler scheme for the λ-equation. Let $\Delta t, \Delta a$ denote the mesh scale in time and space. For (1.1), we have, using standard convention

$$(2.1)\quad\begin{cases}
\dfrac{v^{n+1}_{j+\frac{1}{2}} - v^n_{j+\frac{1}{2}}}{\Delta t} - \dfrac{u^{n+\frac{1}{2}}_{j+1} - u^{n+\frac{1}{2}}_{j}}{\Delta a} = 0 \\[3em]
\dfrac{u^{n+\frac{1}{2}}_{j} - u^{n-\frac{1}{2}}_{j}}{\Delta t} + \dfrac{\overline{P}^{n}_{j+\frac{1}{2}} - \overline{P}^{n}_{j-\frac{1}{2}}}{\Delta t} = 0 \\[3em]
\dfrac{e^{n+1}_{j+\frac{1}{2}} - e^n_{j+\frac{1}{2}}}{\Delta t} + \dfrac{1}{2}\Big(\overline{p}^{\,n+1}_{j+\frac{1}{2}} + \overline{p}^{\,n}_{j+\frac{1}{2}}\Big)\dfrac{v^{n+1}_{j+\frac{1}{2}} - v^n_{j+\frac{1}{2}}}{\Delta t} = 0 \\[3em]
\dfrac{\lambda^{n+1}_{j+\frac{1}{2}} - \lambda^{n}_{j+\frac{1}{2}}}{\Delta t} = r^{n+\theta}_{j+\frac{1}{2}} ,
\end{cases}$$

where $r^{n+\theta}_{j+\frac{1}{2}} = r(\theta p^{n+1}_{j+\frac{1}{2}} + (1-\theta)p^{n}_{j+\frac{1}{2}}, \cdots)$, $0 \leq \theta \leq 1$. Note that we used the energy equation under the form (1.5). Second order accuracy corresponds to the choice $\theta = \frac{1}{2}$.

Such a scheme is now applied to the linearized version of (1.1), that is (1.16), again using the assumption of frozen coefficients. To avoid fractional indexes, we make the shift $i+\frac{1}{2} \to i$, $n+\frac{1}{2} \to n+1$, etc.

We find

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} - \frac{u_{j+1}^{n+1} - u_j^n}{\Delta a} = 0$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{P_j^n - P_{j-1}^n}{\Delta a} = \bar{\mu}\,\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta a)^2}$$

(2.2)

$$\frac{P_j^{n+1} - P_j^n}{\Delta t} + \bar{a}^2\,\frac{u_{j+1}^{n+1} - u_j^{n+1}}{\Delta a} = \bar{b}\,\frac{\lambda_j^{n+1} - \lambda_j^n}{\Delta t}$$

$$\frac{\lambda_j^{n+1} - \lambda_j^n}{\Delta t} = \frac{1}{\tau}\left(\lambda^*_{P_0} P_j^{n+\theta} + \lambda^*_{v_0} v_j^{n+\theta} - \lambda_j^{n+\theta}\right),$$

where $p_j^{n+\theta} = \theta p_j^{n+1} + (1-\theta)p_j^n$, etc., $0 \leq \theta \leq 1$. To carry out a stability analysis for the scheme (2.2), we set

(2.3)
$$v_j^n = V_0\, r^n\, \phi^j = V_0(e^{\omega \Delta t})^n(e^{ik\Delta a})^j$$

and similar relation for u_j^n, p_j^n, λ_j^n. Substituting (2.3) into (2.2), we find a set of linear equations for V_0, U_0, P_0, l_0:

$$\frac{r-1}{\Delta t}\,V_0 - \frac{e^{ik\Delta a} - 1}{\Delta a}\,r\,U_0 = 0$$

$$\left(\frac{r-1}{\Delta t} + \frac{4\bar{\mu}}{(\Delta t)^2}\sin^2 k\,\frac{\Delta a}{2}\right)U_0 + \frac{1 - e^{-ik\Delta a}}{\Delta a}\,P_0 = 0$$

(2.4)

$$\frac{r-1}{\Delta t}\,P_0 + \bar{a}^2\,\frac{e^{ik\Delta a} - 1}{\Delta a}\,r\,U_0 = \bar{b}\,\frac{r-1}{\Delta t}\,l_0$$

$$\frac{r-1}{\Delta t}\,l_0 = \frac{1}{\tau}\,(\theta r + (1-\theta))(\lambda^*_{P_0} P_0 + \lambda^*_{v_0} V_0 - l_0).$$

The characteristic equation associated to this system is obtained by computing its determinant. We find, for $\beta \equiv \sin^2 \frac{k\Delta a}{2}$

(2.5)
$$\frac{r-1}{\Delta t} \left\{ \overline{\tau} \frac{r-1}{\Delta t} \left[\frac{1}{\overline{a}_f^2} \left(\frac{r-1}{\Delta t}\right)^2 + \frac{4\overline{\mu}}{\overline{a}_f^2} \frac{1}{(\Delta a)^2} \beta \frac{r-1}{\Delta t} + \frac{4}{(\Delta a)^2} \beta r \right] \right.$$
$$\left. + (\theta r + (1-\theta)) \left[\frac{1}{\overline{a}_e^2} \left(\frac{r-1}{\Delta t}\right)^2 + \frac{4\overline{\mu}}{\overline{a}_e^2} \frac{1}{(\Delta a)^2} \beta \frac{r-1}{\Delta t} + \frac{4}{(\Delta a)^2} \beta r \right] \right\} = 0,$$

where we used the definition of the frozen and equilibrium mass sound speeds (see the Appendix) and the definition (1.22) of $\overline{\tau}$.

Equation (2.5) is to compare to its continuous analog (1.23). Of course, $r = 1$ is one trivial root. To insure stability, we have to check that the remaining roots of (2.5) do have modulus smaller or equal than 1. This will be achieved assuming conditions on Δt and Δa.

To simplify notations, let us introduce

(2.6)
$$A = 4 \beta \overline{\mu} \frac{\Delta t}{(\Delta a)^2} , \qquad\qquad B = 4 \left(\overline{a}_f \frac{\Delta t}{\Delta a}\right)^2 \beta$$

$$B' = 4 \overline{a}_e \left(\frac{\Delta t}{\Delta a}\right)^2 \beta \qquad\qquad C = \left(\frac{\overline{a}_f}{\overline{a}_e}\right)^2 \frac{\Delta t}{\overline{\tau}} .$$

Therefore (2.5), up to a factor $\frac{r-1}{\Delta t}$, can be rewritten as

(2.7)
$$(r-1) \left[(r-1)^2 + A(r-1) + Br \right]$$
$$+ C(\theta(r-1) + 1) \left[(r-1)^2 + A(r-1) + B'r \right] = 0.$$

We are now looking for conditions insuring that the roots r of (2.7) satisfy $|r| \leq 1$. Classically, we perform the transformation $r = \frac{1+z}{1-z}$ ($|r| \leq 1 \Longleftrightarrow \mathrm{Re}\, z \leq 0$) and apply the Routh-Hurwicz criterium. From (2.7), we deduce

$$(2.8) \qquad \sum_{j=0}^{3} \alpha_j z^j = 0.$$

The coefficients α_j in (2.8) are given by

$$(2.9) \qquad \alpha_3 = 2(4 - 2A - B) + (2\theta - 1)C(4 - 2A - B')$$

$$(2.10) \qquad \alpha_2 = 4A + 2(2\theta - 1)AC + C(4 - 2A - B')$$

$$(2.11) \qquad \alpha_1 = 2B + (2\theta - 1)B'C + 2AC$$

$$(2.12) \qquad \alpha_0 = B'C.$$

The Routh-Hurwicz criterium insures $\mathrm{Re}\, z \leq 0$, for any root z of (2.8), as soon as (see [5])

$$(2.13) \qquad \alpha_2, \alpha_3 \geq 0$$

$$(2.14) \qquad \alpha_1 \alpha_2 - \alpha_0 \alpha_3 \geq 0.$$

By the definitions (2.6) (we recall $\beta \equiv \sin^2 \frac{k\Delta a}{2}$) and the property (1.21), we have

(2.15)   A, B, B´, C $\geq$ 0   and   B $\geq$ B´.

To study (2.13) and (2.14), we discuss according to the sign of $2\theta - 1$ ($0 \leq \theta \leq 1$).

i) The case $2\theta - 1 \geq 0$.

Here the scheme for the λ-equation is unconditionally stable. In view of (2.9),(2.10), and (2.15), the condition (2.13) is satisfied as soon as

(2.16) $4 - 2A - B \geq 0$ and $4 - 2A - B´ \geq 0$.

Due to (2.6) and (1.21), we finally obtain the sufficient condition

(2.17) $(\overline{a}_f \frac{\Delta t}{\Delta a})^2 + 2\overline{\mu} \frac{\Delta t}{(\Delta a)^2} \leq 1$.

Thus, we recover the usual condition of stability for the Von Neumann-Richtmyer scheme (see [9]) but with the appropriate sound speed. Next, we look at the condition (2.14), which reads

$$(4A + 2(2\theta - 1)AC)(2B + (2\theta - 1)B´C + 2AC)$$

(2.18)

$$+ 2AC^2(4 - 2A - B´) + 2(4 - 2A)(B - B´)C \geq 0.$$

Since $2\theta - 1 \geq 0$, (2.18) is satisfied as soon as (2.17); hence, (2.16) holds.

In summary, (2.17) insures (linear) stability for (2.2) if $\frac{1}{2} \leq \theta \leq 1$.

ii) <u>The case</u> $2\theta - 1 \leq 0$.

Here the scheme for the λ-equation becomes explicit. Again using (2.6), we rewrite (2.13) as

(2.19) $\qquad 2(4 - 2A - B - 4(1 - 2\theta)C) + (1 - 2\theta)C(2A + B') \geq 0$

(2.20) $\qquad 2A(2 - (1 - 2\theta)C) + C(4 - 2A - B') \geq 0.$

Thus, due to (2.15), (2.19) is satisfied as soon as

(2.21) $\qquad 4 - 2A - B - 4(1 - 2\theta)C \geq 0.$

With (2.6), we thus obtain the sufficient condition

(2.22) $\qquad (\overline{a}_f \frac{\Delta t}{\Delta a})^2 + 2\overline{\mu} \frac{\Delta t}{(\Delta a)^2} + (1 - 2\theta)(\frac{\overline{a}_f}{\overline{a}_e})^2 \frac{\Delta t}{\overline{\tau}} \leq 1;$

that is the condition (2.17) enforced by an extra term coming from the explicit scheme used for the λ-equation. We remark that this extra term vanishes in the frozen limit $\overline{\tau} \to +\infty$.

Of course, (2.21) implies (2.16), since $B \geq B'$; therefore, (2.20) is satisfied as soon as

(2.23) $\qquad 2 - (1 - 2\theta)C \geq 0,$

and we find the natural condition

(2.24)
$$\left(\frac{\overline{a}_f}{\overline{a}_e}\right)^2 \frac{1-2\theta}{2\overline{\tau}} \Delta t \le 1.$$

It remains to check (2.14), i.e., (2.18). Due to (2.23), it is sufficient to satisfy

(2.25)
$$2B - (1 - 2\theta)B^{\sim}C > 0$$

or

(2.26)
$$\frac{1-2\theta}{2\overline{\tau}} \Delta t \le 1.$$

But, since $\overline{a}_f^2 \ge \overline{a}_e^2$, (2.26) is a consequence of (2.24).

In summary, (2.22) and (2.24) insure (linear) stability for (2.2), if $0 \le \theta \le \frac{1}{2}$.

The above results could be easily generalized to similar 2-D reactive flows discretized by the natural extension of the Von Neumann-Richtmyer scheme (see [6], for instance). Similar results are anticipated for the Godunov scheme, which reduces, after linearization, to the Courant-Isaacson-Rees scheme ([7]). They will be published elsewhere.

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APPENDIX

Here, for the convenience of the reader, we recall some classical results concerning the definition of the sound speed in reactive flows (see for instance [3],[8],[1]). Here we take $\overline{p} = p$, that is, (see (1.2)); we suppose $\mu = \overline{\mu} = 0$.

According to the laws of thermodynamics,

$$(A.1) \qquad\qquad TdS = de + pdv - \nu d\lambda,$$

where T, S, ν are the temperature, the entropy, and the chemical potential. From (A.1) and (1.3), we deduce

$$(A.2) \qquad Tds = \frac{\partial e}{\partial p}\, dp + (\frac{\partial e}{\partial v} + p)dv + (\frac{\partial e}{\partial \lambda} - \nu)d\lambda.$$

Now, we consider two cases.

1) **The flow is frozen:** $dS = d\lambda = 0$.

Then, (A.2) becomes

$$(A.3) \qquad\qquad (\frac{\partial p}{\partial v})_{s,\lambda} = \frac{\frac{\partial e}{\partial v} + p}{\frac{\partial e}{\partial p}}$$

or

$$(A.4) \qquad\qquad (\frac{\partial p}{\partial \zeta})_{s,\lambda} = v^2\, \frac{\frac{\partial e}{\partial v} + p}{\frac{\partial e}{\partial p}} \equiv c_f^2\, .$$

Therefore, the definition (1.9) of $\bar{a}^2$ (where $\bar{p} = p$) is that of the frozen mass sound speed $\rho^2 c_f^2$. We shall use the standard notation $a_f^2 \equiv \rho^2 c_f^2$.

ii) <u>The flow is in chemical equilibrium</u>: $dS = 0$ <u>and</u> $\lambda = \lambda^*(p,v)$.

In particular $\nu = 0$ and $d\lambda = \dfrac{\partial \lambda^*}{\partial p} dp + \dfrac{\partial \lambda^*}{\partial v} dv$ in (A.2); collecting terms we find

(A.5)
$$\left(\frac{\partial p}{\partial v}\right)_{S,\lambda=\lambda^*(p,v)} = - \frac{\dfrac{\partial e}{\partial v} + p \dfrac{\partial \lambda^*}{\partial v} \dfrac{\partial e}{\partial \lambda}}{\dfrac{\partial e}{\partial p} + \dfrac{\partial \lambda^*}{\partial p} \dfrac{\partial e}{\partial \lambda}} .$$

But $\left(\dfrac{\partial p}{\partial v}\right)_{s,\lambda=\lambda^*} = - \dfrac{1}{v^2} \left(\dfrac{\partial p}{\partial \rho}\right)_{s,\lambda=\lambda^*}$, and thus we find the equilibrium sound speed:

(A.6)
$$c_e^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_{s,\lambda=\lambda^*} = v^2 \frac{\dfrac{\partial e}{\partial v} + p \dfrac{\partial \lambda^*}{\partial v} \dfrac{\partial e}{\partial \lambda}}{\dfrac{\partial e}{\partial p} + \dfrac{\partial \lambda^*}{\partial p} \dfrac{\partial e}{\partial \lambda}} .$$

The equilibrium mass sound speed is then $a_e^2 \equiv \rho^2 c_e^2$, and we easily check (see (1.20),(1.8),(1.10)) that

(A.7)
$$\frac{\bar{a}^2 - \bar{b} \lambda^*_{v_0}}{1 - \bar{b} \lambda^*_{p_1}} = a_e^2.$$

Similarly, we find (see (1.20)) that

(A.8)
$$\frac{\bar{a}^2}{\bar{a}^2 - b\lambda_0^*} = \frac{\frac{\partial e}{\partial v} + p}{\frac{\partial e}{\partial v} + p + \frac{\partial \lambda}{\partial v}^* \frac{\partial e}{\partial \lambda}} \ .$$

Finally, in view of (A.4) and (A.6), we have

(A.9)
$$a_f^2 \geq a_e^2 .$$

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16. Abstract

We study a simple model of compressible reacting flow. First, we derive a dispersion relation for the linearized problem, making a distinction between frozen and equilibrium sound speed. Second, we study the stability of the Von Neumann-Richtmyer scheme applied to this model. One finds a natural generalization of the C.F.L. condition.

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